# ADDITIONAL MATHEMATICS

**Paper 0606/12**
**Paper 12**

## Key messages

Candidates should ensure that each question is read carefully and answers given in the required form (**Questions 1**, **5(b)** and **8**). Candidates should also check that the data they are given has been used correctly (**Question 9(b)**). An awareness of the amount of work required for a particular mark allocation would also be useful and can be obtained through practice of past questions. It is also essential that candidates take note of instructions given indicating a required method, as in **Question 5(b)** where candidates were required to make use of their graph. Candidates should also ensure that their work is written clearly and carefully so that misreading errors in their own work are not made (**Question 5(d)**).

## General comments

There were many well presented and highly scoring scripts, showing a very good understanding by candidates of the syllabus objectives. Most candidates were clearly well prepared. It was pleasing to see that candidates made use of supplementary sheets when they needed extra room or wished to make another attempt at a question. There did not appear to be any timing issues.

## Comments on specific questions

### Question 1

Most candidates provided a completely correct solution, with the majority choosing to form a quadratic equation representing the possible points of intersection of the straight line and the curve. Correct use of the discriminant of this quadratic equation usually resulted in a correct exact answer. Most candidates gave their responses in exact form as requested in the question. Candidates who attempted to equate the gradient function of the curve and the gradient of the straight line were less successful as a correct method was often not used.

### Question 2

**(a)** There were two possible approaches to this part of the question, with each approach being equally popular and equally successful. Incorrect solutions were rarely seen. Some candidates chose to differentiate using the product rule. Rather than use the result which clearly had a common factor of $x + 2$ and factorise the derivative, many candidates chose to expand out their result and then factorise. Other candidates chose to expand out the given expression in terms of $x$ and differentiate the result. Very few errors were made.

**(b)** Many candidates recognised the correct form of the cubic curve with a stationary point on the negative $x$-axis and a maximum point in the first quadrant. Some candidates did not realise that the curve only touched the negative $x$-axis. The first part of the question was intended to help candidates determine the detailed shape of the curve. Candidates who sketched a quadratic curve were unable to gain any marks.

**(c)** Again, the first part of the question together with the subsequent sketch, were intended to guide candidates. Many correct responses were seen, with candidates realising that they needed to calculate the $y$-coordinate of the stationary point in order to progress. It was evident however, that some candidates did not understand the demand of the question.

**Question 3**

Most candidates were able to use the binomial expansion correctly and obtain the correct terms in $x^8$ and $x^6$. Some candidates chose to calculate quite a few terms in the expansion, while the more confident candidates were able to identify the relevant terms straight away. There were inevitably sign errors when dealing with the multiplication by $1 - x^2$, but most candidates had an appreciation of the method that needed to be used. Apart from sign errors, the other main error was not dealing with the coefficient of 2 correctly when using the binomial expansion.

**Question 4**

**(a)** Many completely correct solutions were seen. However, some candidates chose to give their final answer as $\lg \dfrac{\frac{x^3}{y^4}}{1000}$. It was expected that a final answer of $\lg \dfrac{x^3}{1000y^4}$ be given. Too many candidates incorrectly simplified their answer to $\lg \dfrac{1000x^3}{y^4}$, not appreciating the correct use of the negative signs in the last two terms or not being able to simplify fractions correctly. Candidates who were unable to deal with the term of 3 and write it as $\lg 1000$ were able to gain credit for appropriate use of the power rule and division rule for logarithms.

**(b)** Most candidates realised that a change of base was necessary in order to make progress. Most chose to work with base 3 logarithms, but those who chose to work with base $x$ logarithms were usually equally successful provided a correct method was used. A few candidates chose to use either base 10 logarithms or natural logarithms. These candidates were usually less successful as extra calculations involving these logarithms were needed. The majority of candidates formed a quadratic equation in the logarithm of their choice, solved it and obtained the correct answers.

**Question 5**

**(a)** Very few candidates could not calculate the correct coordinates and obtain a straight line graph. Occasionally, an incorrectly plotted point or points would result in an accuracy mark not being awarded.

**(b)** It was important that candidates take note of the instruction in the question to use their graph. It was intended that candidates make use of a transformed straight line equation. Equating the gradient of the straight line plotted in **part (a)** meant that ln *b* and hence *b* could be calculated. Use of this value of *b*, or use of the intercept with the *y*-axis of the straight line graph in **part (a)** enabled candidates to find the value of ln *A* and hence *A*. Variations of these approaches were acceptable.

The question was intended to test the syllabus area dealing with transformations to straight line form, hence the instruction to make use of the straight line graph drawn. It was not intended that use of the original data in the equation $y = Ab^{x^2}$ together with the solution of the resulting simultaneous equations be made.

Many candidates also ignored the instruction to give their answers correct to 1 significant figure, highlighting the need to read the question carefully and give final answers in the form demanded.

Candidates should also consider the appropriateness of their answers. For example, some candidates obtained a value of 1 for *b*, which is clearly unrealistic.

**(c)** It was essential that a correct method be chosen by candidates. Either use of a correct equation in terms of logarithms, use of the original equation $y = Ab^{x^2}$ or use of the original straight line graph were acceptable, provided the values of *A* and *b* were used appropriately. Candidates who had obtained incorrect values for *A* and *b* were able to obtain a method mark if their incorrect values had been used in a correct method. Solutions from using the graph in **part (a)** were not common even though the results were more reliable than those calculated by some candidates. The graph could have been used as a check of calculated results.

**(d)** The same principles applied in **part (c)** were also applied in this part of the question. There were more errors in this part, mainly due to poor arithmetic or misreading of badly written equations. For example, the equation $20 = \frac{1}{2}\left(4^x\right)$ was sometimes written as $10 = 4^x$ or $20 = 2^x$. An example of misreading their own work was to write $4x$ rather than $4^x$.

**Question 6**

Many completely correct responses were seen, showing that most candidates had a good understanding of integration and the inclusion of arbitrary constants. Very few candidates omitted arbitrary constants and subsequently found their values. Most errors involved arithmetic slips or errors in the coefficient of the integrated terms.

**Question 7**

**(a) (i)** Very few incorrect responses were seen.

**(ii)** Candidates should be guided by mark allocation and in this case, the size of the answer space. The allocation of 2 marks implies that a simple calculation is required. In this case, it was intended that candidates calculate the number of passwords that did not contain a symbol and subtract it from their answer to **part (i)**. Although there were alternative methods to this problem, very few candidates used them successfully as they did not consider all the possible arrangements.

**(b)** Many correct solutions were obtained although often several attempts had been made in order to obtain an integer answer. Most candidates were able to simplify the numerical parts of the equation (obtaining 16 and 12 or equivalent) having written down a correct equation in terms of factorials. Many candidates were unable to simplify the factorial terms correctly, highlighting a need to practice manipulation of factorials.

**Question 8**

This question was intended to test the problem-solving skills of candidates. There were many completely correct responses, together with many responses gaining over half marks. Candidates needed to calculate the coordinates of the points *A*, *B* and *C*. Most were successful at this, although there were occasional arithmetic slips and also algebraic slips when calculating the x-coordinate of *B*. Unfortunately, errors in these first few calculations meant that there would be subsequent errors when calculating the required area. The easiest method was to calculate the area of the triangle formed by the given straight line and the perpendicular through the point *B*. This could then be added to the area under the curve between the point *A* and the point where the perpendicular from *B* met the *x*-axis. There were other appropriate methods, but often the complete area was not calculated. Most candidates were able to integrate correctly and obtain method marks if incorrect limits had been used due to initial errors. Many candidates chose to use integration to find the area of the triangle and this was also acceptable. A few candidates attempted to use areas enclosed by the *y*-axis, but with little success. Some candidates were unable to gain marks as they resorted to the use of a calculator rather than give an exact form as required by the question. It was again important the form of the final answer be checked as answers of $\frac{11}{8} - \ln\frac{64}{27}$ although correct, were not in the form required.

**Question 9**

**(a) (i)** Very few incorrect answers were seen.

**(ii)** Some candidates were unable to identify the part of the given vector equation which yielded the velocity vector. Of those that did, most were able to find the speed as required.

**(iii)** Most candidates made a reasonable attempt at this part of the question, with many showing that by equating like vectors, two different times were obtained thus showing that the particle did not pass through the given point. Others chose to find one time and then show that the position vector at this time was not the same as the given position vector. There were some simple arithmetic errors and some candidates, although having obtained two different values for time, still concluded that the particle passed through the given point.

**(b)** Many correct solutions were seen, showing a good understanding of vector geometry. Most errors occurred when candidates misread the given ratio as $AB : BC = 1 : 4$. This again highlights the need for candidates to read the question carefully and not make assumptions.

**Question 10**

**(a)** Few completely correct solutions were seen. It was intended that candidates isolate the trigonometric terms, (e.g. $\cos\theta = x + 2$ and $\sin\theta = \frac{2}{y}$) and use an appropriate trigonometric identity, (e.g. $\cos^2\theta + \sin^2\theta = 1$) to eliminate them. Other methods were acceptable as long as correct identities and eventual eliminations were achieved.

**(b)** Many completely correct solutions were seen with the majority of candidates obtaining at least 4 marks. Most dealt competently with the half angle. Many candidates did not obtain full marks as they were unable to obtain the negative angle solutions.

# ADDITIONAL MATHEMATICS

**Paper 0606/22**
**Paper 22**

## Key messages

In order to succeed in this examination, candidates need to read each question carefully and ensure that they have identified key statements and information. Candidates also need to show sufficient method so that marks can be awarded. Candidates need to take care that their calculator is in the appropriate mode when working with trigonometric expressions. Attention should be given to the instructions on the front page of the examination paper. Candidates should ensure that their answers are given to at least the accuracy required in a question. When no particular accuracy is asked for, candidates should ensure that they follow the instructions on the front page. Candidates should also be familiar with, and check, the formulae given on page 2 of the examination paper.

## General comments

Most candidates were able to recall and apply techniques in order to solve problems when needed. Many candidates attempted complete solutions, with sufficient working shown to gain full credit.

Candidates needed to be aware that notation should be correct and expressions unambiguously stated. For example, the domain of a function with argument $x$ should be stated in terms of $x$. This was required in **Question 8(a)(i)**. Use of brackets or correct ordering of terms in a product should be used to ensure that terms cannot be misinterpreted. This was required in **Question 9(b)** in this examination.

It was expected that candidates would have a good understanding of how to use ratio. Many candidates demonstrated this in **Question 6(b)** and **Question 9(a)** in this examination.

Candidates who did not show full method because they used their calculator to perform key operations, such as finding the value of the derivative of a function for a particular value, without finding an expression for the derivative, did not gain full credit. This was seen in **Question 7(a)** and **Question 11** in this examination.

Candidates usually presented their work in a clear and logical manner. Some candidates used additional paper. Candidates who did this usually added a comment in the answer space in their main script to indicate that their answer was written, or continued, elsewhere. This was very helpful to examiners in ensuring that all work was credited.

Most candidates attempted to answer all questions. Candidates seemed to have sufficient time to attempt all questions within their capability.

## Comments on specific questions

### Question 1

This was an accessible start to the paper for many candidates. A good proportion of graphs were fully correct. Many candidates drew the graph of $y = 4\cos 2x$ and then reflected the relevant section in the $x$-axis. This proved to be the most successful approach. A few candidates omitted to draw cusps. Other candidates had sections that were either straight or of incorrect curvature. The question required candidates to mark the intercepts with the axes. The majority of those who attempted to draw a graph of correct shape marked these correctly. Only on occasion were intercepts omitted or incorrect.

**Question 2**

The majority of candidates squared the fraction as an initial step and then went on to rationalise the denominator. A good proportion showed sufficient method to demonstrate that they had not used a calculator and earned all the marks available. A few candidates did not show sufficient terms when expanding to justify the denominators being $13-4\sqrt{3}$ in the first step or 121 when rationalised and this was penalised. A few candidates made sign or arithmetic slips or made slips in dealing with *x*. The few candidates who rationalised first and then squared were also reasonably successful, although those squaring the numerator in the form $2\sqrt{33}x+x\sqrt{11}$ occasionally made arithmetic slips.

**Question 3**

There were two main approaches to this question. Squaring both sides of the inequality and solving the quadratic equation formed to find the critical values, was the slightly more popular method. This was, perhaps, the more straightforward method of arriving at the correct final solution as it was easier to determine the correct form of the answer from this approach. Occasionally candidates made sign or arithmetic slips using this method but they were otherwise very successful. Candidates who chose to form and solve two linear equations were often able to find the correct critical values but many were unable to form the correct final inequality. A common incorrect answer using this method was $x \leqslant -\frac{7}{3}, x \leqslant -\frac{1}{7}$.

**Question 4**

Most candidates were able to show that the given expression for *y* could be written as $\sin 5x$. Only a few candidates made slips in doing this. The most efficient method was to use the trigonometric identity given on page 2 of the examination paper to deduce that the numerator was 1. A good proportion of candidates correctly integrated $\sin 5x$ and went on to offer a fully correct solution. A few candidates offered either $-5\cos 5x$ or $-\cos 5x$ or $\frac{1}{5}\cos 5x$ and most of these were able to earn the method mark for the correct substitution of the limits. Most candidates showed the substitution of the upper and lower limits into their expression. This was good practice. A small number of candidates thought that the cosine of 0 was 0 and did not show the substitution of the lower limit into their expression. This error was not condoned. A few other candidates made slips with minus signs and arrived at the answer 0.

**Question 5**

**(a)** Almost all candidates offered a correct solution. The simplest method was to use the factor theorem to show that 1 – 2 – 19 + 20 = 0 and this approach was used by nearly all candidates.

**(b)** Again, this part of the question was very well answered with almost all candidates able to derive the correct quadratic factor and factorise this correctly to form the product of linear factors. A few sign slips were made but these were quite rare. A small number of candidates omitted to factorise the quadratic factor. No calculator was to be used in this question and candidates needed to use the given factor $x-1$ in their solution. This was indicated by the use of the word 'Hence' in this part. Therefore, candidates who did not find the quadratic factor were penalised.

**(c)** In this part of the question, candidates needed to deduce that $x = e^y$ and then solve. Again, this was well answered and most candidates rejected $e^y = -4$. Very few candidates offered a value from this but, in the case of those that did, it was commonly –ln 4. Some candidates wrote ln 5 as 1.61, or similar, and this was not acceptable as calculators were not to be used in this question. The weakest responses offered $x = e^{3y}$ or similar.

**Question 6**

**(a) (i)** This part of the question was very well answered and very few errors were seen.

**(ii)** Again, this part of the question was very well answered. On occasion, candidates used a power of 9, rather than 10, in the sum formula. These candidates may have improved if they had checked the formulae given on page 2 of the examination paper. A few candidates rounded their answer to 3 significant figures without stating the exact answer first. This was not accepted. Candidates should be aware that fractions or terminating decimals are exact values and therefore do not need to be rounded. Candidates need to understand that it is good practice to state the exact value before rounding, if they are unsure whether rounding is appropriate.

**(iii)** Almost all candidates were able to state the correct answer for this part of the question.

**(b)** A good proportion of candidates offered fully correct solutions. Candidates who used the given equation of sums and went on to simplify, quickly found the value of $d$. A few candidates did not simplify immediately, but formed an equation using the ratio of terms, that was also given, and solved simultaneously. These candidates were often, also, successful with only occasional sign or arithmetic slips seen. For example, candidates who divided the sum equation through by 10 often forgot to divide 400 by 10. A few candidates either omitted to find the sum of the first three terms or simply listed the terms without summing them. Some candidates misinterpreted the ratio and the most common misinterpretation in this case was to write $a = 5(a + 5d)$. Weaker responses either misused the ratio as $a = 1$ and $a + 5d = 5$ or used formulae appropriate for a geometric progression.

**Question 7**

**(a)** Most candidates followed the instruction to differentiate. A few candidates tried to simplify the given trigonometric fraction before differentiating. Often this was successful but not all candidates who used this approach were accurate. The resulting expressions were not usually simpler than the given form and taking this approach introduced the opportunity of making an unnecessary error.

Many found that differentiating $1 + \cos^2 x$ was beyond them, however. It was common to see this differentiated as either $-2\sin^2 x$ or $-\sin^2 x$ or $-2\sin x$. Most candidates were able to differentiate $\tan x$ as $\sec^2 x$, although on occasion $\sec x$ was stated for this. The majority of candidates used a correct structure for the quotient rule. Some candidates attempted to simplify their derivative prior to substituting $\frac{\pi}{4}$. This sometimes resulted in errors and loss of accuracy. When the derivative was not shown correctly at any stage, evidence of substitution of $\frac{\pi}{4}$ into the candidate's derivative needed to be seen. Candidates should be aware that this is a key step in the method and should be shown. Some candidates were clearly using their calculator to find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1+\cos^2 x}{\tan x}\right)\bigg|_{x=\frac{\pi}{4}}$ as $\delta y = -4h$ was commonly seen following an incorrect derivative. This was not accepted. Some candidates had their calculator in degree mode. It is expected that candidates understand that, for the standard derivatives for trigonometric functions to be correct, the unit must be radians. Weaker responses sometimes used of $\frac{\pi}{4} + h$ instead of $\frac{\pi}{4}$ or instead of $h$.

**(b)** Many candidates were able to find correct expressions for the first and second derivative. Many of these candidates went on to provide a completely correct solution. A few candidates used the quotient rule, rather than the chain rule, and these sometimes made errors. Commonly 1 was differentiated as 1 resulting in an incorrect first derivative, or the first derivative was not simplified and the second derivative became quite complex, resulting in errors. These candidates may have improved if they had thought about what the simplest approach to finding the derivative was considering that the numerator of the fraction was a constant. Candidates who applied knowledge of indices and used the chain rule were almost always correct. In this case, errors were not common. When an error was made it was usually adding 1 to the negative power, rather than subtracting 1. Most candidates manipulated the algebraic fractions correctly and derived the correct form. A few candidates made sign or arithmetic errors and this was more common when candidates did not use the lowest common multiple of the denominators they were combining. The question required candidates to arrive at the given form, $\frac{(x+1)(x-4)}{(x-3)^5}$. Sufficient evidence of working towards the given answer, and not back from the given answer, needed to be seen.

**Question 8**

**(a) (i)** Most candidates found this to be quite challenging. Fully correct solutions were not common. Some candidates offered partially correct solutions. Often these were $x \geqslant 3$ or $x < 5$. A few candidates did find the limits of the range but combined them in an incorrect form or only stated them as the range of f, for example. A large proportion of candidates omitted to use the range of f and attempted to find an expression for the inverse function here. These candidates often stated the domain as $x > 5$.

**(ii)** Again, candidates found this part of the question to be challenging. The most efficient method of solution was to understand and use $\mathrm{f}\,\mathrm{f}^{-1}(x) = x$ or $\mathrm{f}^{-1}(x) = \mathrm{f}^{-1}\left(\sqrt{5x-4}\right)$ to form and solve the equation $x = \sqrt{5x-4}$. This was not often seen, however. Again, it was far more common for candidates to form or use the inverse function. This was fairly successful but there was a greater likelihood that a sign error would be made. Candidates who were successful as far as finding $x = 1$ and $x = 4$ often omitted to reject 1 which was outside the domain.

**(iii)** The best responses were neat, accurate and carefully drawn over the correct domain, with approximately equally spaced scales on the axes, and all key features labelled as required. Many candidates drew lines to represent the asymptotes and this was good practice. Some candidates did not use the correct domain, including values of $x$ that were less than 3 but many of these candidates were otherwise correct. A few candidates drew correct graphs for $y = \mathrm{f}(x)$ but did not make a simple reflection in $y = x$ for the inverse function. Instead they drew $y = \mathrm{f}^{-1}(x)$ using the rule for the inverse function they had previously found. This was rarely successful. Some candidates drew incorrect graphs for $y = \mathrm{f}(x)$. Commonly these were attempts at $k + 2\mathrm{e}^{x}$ or $k + 2\mathrm{e}^{-x}$, for example. Many of these candidates were able to earn a mark for the correct reflection of this graph in the line $y = x$. Those who were completely incorrect usually reflected their attempt at the graph of $y = \mathrm{f}(x)$ in the $x$-axis.

**(b)** It was in this part of the question that candidates were expected to find and use an expression for $\mathrm{f}^{-1}(x)$. Many candidates were able to find a correct expression for the inverse function and a good proportion of these went on to state a correct, simplified form of the composite function. Some candidates made sign errors when finding the inverse function and a few other candidates made method errors, such as $\mathrm{e}^{-y} = \dfrac{x-5}{-2}$ becoming $-y = -\ln\dfrac{x-5}{2}$. A very good number of candidates used a correct order of composition. A few candidates were unable to simplify the argument of the logarithm and the 2 was commonly mishandled. A very small number of candidates either found an expression for $\mathrm{gf}^{-1}(x)$ or formed a product of functions.

**Question 9**

**(a)** This question was well answered with many candidates earning full marks. A few candidates omitted to include a section of the perimeter in their answer. A few other candidates were unable to interpret the ratio correctly. In this case it was common for the ratio to be treated as $OA : AC$ rather than $OA : OC$. A very small number of candidates found a difference of areas. These candidates may have improved if they had read the question more carefully.

**(b)** Candidates found this part of the question to be quite challenging. The simplest method of solution was to find the difference of the area of the circle and the area of the sector $PRQ$. Many candidates appreciated this. In order to give their answer in terms of $a$ and $\phi$, candidates needed to find angle $PRQ$ and an expression for $y$ in terms of $a$ and $\phi$. A good number were able to state or indicate the correct sector angle but fewer were able to find an expression for $y$. Some attempts involved trigonometry and some of these were correct. Some candidates used correct right-angled triangle trigonometry to find $y = 2a\cos\phi$. This was possibly the simplest expression for $y$. Other candidates used an incorrect trigonometric ratio. Candidates who attempted the sine or cosine rule were also successful, although the most common problem with these methods was to have an incorrect angle $POR$. The most common issue was to use $180 - 2\phi$ rather than $\pi - 2\phi$ and this was not condoned. Candidates who attempted the difference of areas sometimes omitted to square their expression for $y$. A commonly seen incorrect method was to attempt to sum two segments and sector $POQ$ and subtract this sum from the area of the circle.

**Question 10**

**(a) (i)** This part of the question was reasonably well answered. Most candidates integrated $6t$, added a constant of integration and used the information given to find an expression for the velocity in terms of $t$. These candidates almost always went on to find the correct velocity at time $t = 3$. A few candidates omitted the constant of integration. This was not condoned. A few other candidates attempted to use the exponential expression for $a$ when $t \geqslant 3$ in this part or attempted to use *suvat* equations in some way.

**(ii)** This part of the question was also reasonably well answered with a good number of candidates giving a fully correct solution. The issues in this part of the question were very similar to those in **part (i)**. A few candidates, having found the correct velocity in **part (i)**, integrated only $3t^2$ rather than $3t^2-1$. This was not condoned.

**(b)** This part of the question was more challenging. Good responses often combined the powers of e as an initial step and included the constant of integration at each stage. A reasonable number of fully correct solutions were seen. Some candidates correctly found an expression for $v$, including the constant of integration, but then incorrectly used $t = 1$ and $v = 2$ to find the constant. A few candidates omitted the constant of integration in the first step. Other candidates were unable to integrate correctly at any stage. Common errors in weaker responses were to raise the power of e by 1 and divide by the new power, to integrate $e^3$ as $\frac{e^4}{4}$ or to include a natural logarithm.

**Question 11**

A reasonable number of fully correct solutions were seen. To be successful in this question, candidates needed to complete several steps. Initially, they needed to solve $\sin(4x - \pi) = 0$ to find the value of $x$ that was between $\frac{\pi}{2}$ and $\pi$. A reasonable number of candidates found the correct value, $\frac{3\pi}{4}$, but very many stated a different multiple of $\frac{\pi}{4}$ for $a$, most commonly $\frac{\pi}{4}$ itself. A good number of candidates were able to differentiate $\sin(4x - \pi)$ successfully. Many of these were able to use their value of $a$ correctly to find the gradient of the tangent and then the normal. A small number of candidates did not state a derivative and were penalised for this, as indicated in the rubric on the front of the examination paper. A few candidates incorrectly found the gradient at the point $x = 0$ rather than $x = a$. This was not condoned. A good number of candidates were able to form an equation for the normal using their values. Most, although not all, candidates working with correct values were able to identify the coordinates of $B$ and went on to complete the solution correctly. A few candidates did not give the correct answer in exact form. These candidates commonly either omitted the exact form altogether or omitted the square from $\pi$ in the numerator.